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A RELATIONSHIP FOR THE WALL EFFECT ON THE SETTLING VELOCITY OF A SPHERE AT ANY FLOW REGIME

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Abstract—By using a fluid dynamic analogy between a single particle in a tube and a multiparticle suspension previously derived, a relationship for the wall effect on the terminal settling velocity of a single particle in a cylindrical tube is obtained. Unlike previous relationships, this one is valid for any flow regime, from viscous to fully inertial, and it is in good agreement with experimental evidence. Copyright © 1996 Elsevier Science Ltd.

Key Words: terminal settling velocity, wall effect, viscous flow, inertial flow

1. INTRODUCTION

The retarding effect of the wall container on the single sphere settling velocity has been known and studied, both theoretically and experimentally, for centuries (Newton 1687; Munroe 1888; Ladenburg 1907). It is brought about by the upwards counterflux of fluid which balance the downwards flux of the solid and that of the dragged down fluid; the smaller the area available for the counterflux, i.e. the smaller the container cross section area compared to the particle size, the more important the phenomenon is.

In the creeping flow regime, both fully theoretical and empirical correlations have been proposed regarding the ratio of u_t, the bounded terminal velocity, to $u_{t\infty}$, the unbounded terminal velocity: the empirical equation of Francis (1933)

$$\frac{u_t}{u_{t\infty}} = \left(\frac{1-\lambda}{1-0.475\lambda}\right)^4 \quad [1]$$

where λ is the particle to the tube diameter ratio, stands out for its simplicity and accuracy when compared with experimental evidence. Equation [1] is valid for the whole range of λ and produces predictions on the terminal velocity practically coincident with the more complicate relationship derived theoretically by Haberman & Sayre (1958), as shown by Clift *et al.* (1978) and by Iwaoka & Ishii (1979).

For systems possessing a terminal Reynolds number outside the viscous flow regime, theoretical analysis are no longer available; and experimental data are not abundant either. Fidleris & Whitmore (1961), however, reported, in the most comprehensive experimental investigation published so far (based on some 3000 experimental observations), the effect of the column diameter on the single particle settling velocity for a wide range of $Re_{t\infty}$, 0.054–20000. They found that the phenomenon was less and less important as the Reynolds number increased; their data followed [1] for the lowest Reynolds number systems and, for the highest $Re_{t\infty}$ were satisfactorily described by the Munroe (1988) empirical relationship

$$\frac{u_t}{u_{t\infty}} = 1 - \lambda^{1.5}. \quad [2]$$

For the intermediate flow regime Fidleris & Whitmore (1961) presented their observation in a graphical manner, reproduced here in figure 1, concluding that "*it is difficult to derive a single* *relationship to account for the change in the interference effect of the vessel wall on a falling sphere which occurs with increasing Reynolds number*".

The aim of this paper is to resolve this difficulty as, so far, no general relationships appear to have been proposed capable of covering the whole range of Reynolds numbers (see, e.g. Clift *et al.* 1978).

2. THE BACKGROUND

In a companion paper (Di Felice 1996) a fluid dynamic analogy between a single sphere in a cylindrical tube and a multiparticle solid–fluid suspension has been proposed and we will shortly report here the findings of that paper related to the present work. The analogy is based on a geometrical equivalence between the suspension voidage, ϵ, and λ

$$\epsilon = \frac{1-\lambda}{1-0.33\lambda} \qquad [3]$$

and also on an equivalence between wall function, $f(\lambda)$ which takes into account the increase in drag force on a single sphere in tube due to presence of the wall, and voidage function, $g(\epsilon)$ which takes into account the increase in drag force on a particle in a suspension due to the presence of neighbouring particles,

$$f(\lambda)C_{\mathrm{Dt}} = g(\epsilon)C_{\mathrm{D0}}\epsilon^2. \qquad [4]$$

In [4], C_{Dt} is the single particle drag coefficient calculated at a velocity u_t, C_{D0} is the same coefficient at a velocity u_0, the two velocities being related by $u_t = u_0/\epsilon$.

In the previous paper we wrote the drag force on sphere as

$$F_{\mathrm{D}} = kC_{\mathrm{D}}\,\mathrm{Re}^2 \qquad [5]$$

where k is a constant for a given system.

The drag force on a sphere in terminal condition on an infinite expanse of fluid is then

$$F_{\mathrm{Dt}\infty} = kC_{\mathrm{Dt}\infty}\,\mathrm{Re}_{t\infty}^2. \qquad [6]$$

The same sphere falling in terminal condition in a tube will experience a drag $f(\lambda)$ times that of a sphere falling at the same velocity in an infinite expanse

$$F_{\mathrm{D}} = f(\lambda)kC_{\mathrm{Dt}}\,\mathrm{Re}_t^2. \qquad [7]$$

As in both cases the particle is in terminal condition, the drag force in [6] and [7] must be equal. By equating the two equations, the ratio of bounded to unbounded terminal settling velocity is expressed function of the wall function as

$$\frac{u_t}{u_{t\infty}} = \left(\frac{C_{\mathrm{Dt}\infty}}{C_{\mathrm{Dt}}f(\lambda)}\right)^{0.5}. \qquad [8]$$

Equation [8] becomes, with the introduction of the analogy between wall and voidage function, [4],

$$\frac{u_t}{u_{t\infty}} = \left(\frac{C_{\mathrm{Dt}\infty}}{C_{\mathrm{D0}}\epsilon^2 g(\epsilon)}\right)^{0.5}. \qquad [9]$$

3. THE DERIVATION OF THE NEW RELATIONSHIP

In [9] the ratio between bounded and unbounded terminal settling velocities can be numerically evaluated in a straightforward manner, as all the quantities appearing on the right hand side of the equation are easily obtainable.

For the single particle drag coefficients various empirical correlations covering the whole range of Reynolds number are in use (we will utilize here the simple one of Dallavalle 1948), whereas the voidage function is known for any flow regime, having been derived from experimental observation on the behaviour of fixed and fluidised suspensions, and is given by

$$g(\epsilon) = \epsilon^{-\beta} \qquad [10]$$

where β is a weak function of the suspension Reynolds number (Di Felice 1994)

$$\beta = 3.7 - 0.65 \exp\left(-\frac{(1.5 - \log(\mathrm{Re}_0))^2}{2}\right). \qquad [11]$$

From [9]–[11] the wall correction for the terminal settling velocity assume extremely simple functions for both the viscous flow regime

$$\frac{u_t}{u_{t\infty}} = \epsilon^{2.7} = \left(\frac{1-\lambda}{1-0.33\lambda}\right)^{2.7} \qquad [12]$$

(where the accidental similarity with the Francis equation is worth noting) and the inertial flow regime

$$\frac{u_t}{u_{t\infty}} = \epsilon^{0.85} = \left(\frac{1-\lambda}{1-0.33\lambda}\right)^{0.85}. \qquad [13]$$

Equations [12] and [13] are compared with the correspondent empirical relationships, [1] and [2], in figure 2: in both cases the agreement is quite good.

For the intermediate flow regime no direct analytical relationship can be obtained from [9]. However, when such expression is plotted in logarithm coordinates at Re_t constant, as shown in figure 3, a reasonably good straight line is obtained for all the cases. This is to say that [9] is practically equivalent to the simpler

$$\frac{u_t}{u_{t\infty}} = \epsilon^{\alpha} = \left(\frac{1-\lambda}{1-0.33\lambda}\right)^{\alpha} \qquad [14]$$

with α a function of Re_t. Values of α were obtained, with the help of diagrams as figure 3, for a discrete number of Re_t, then fitted with a sigmoidal shape function over the whole range of

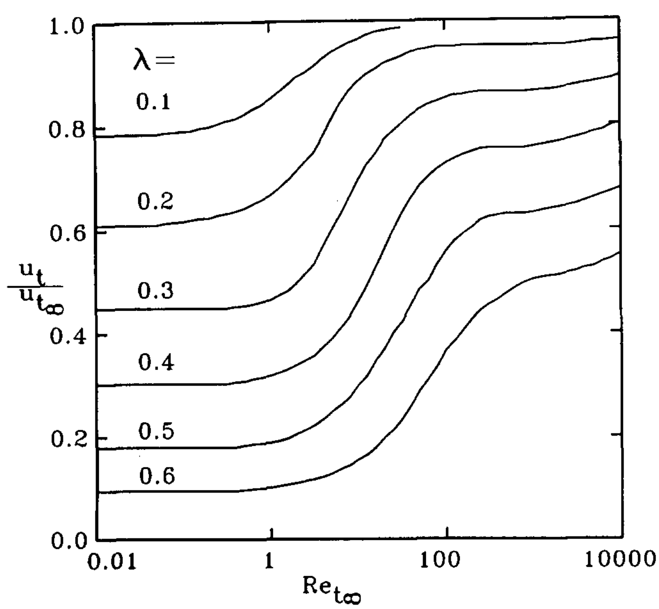

Figure 1. The ratio of bounded to unbounded terminal settling velocities as experimentally found by Fidleris & Whitmore (1961) for a wide range of Reynolds number and selected values of λ.

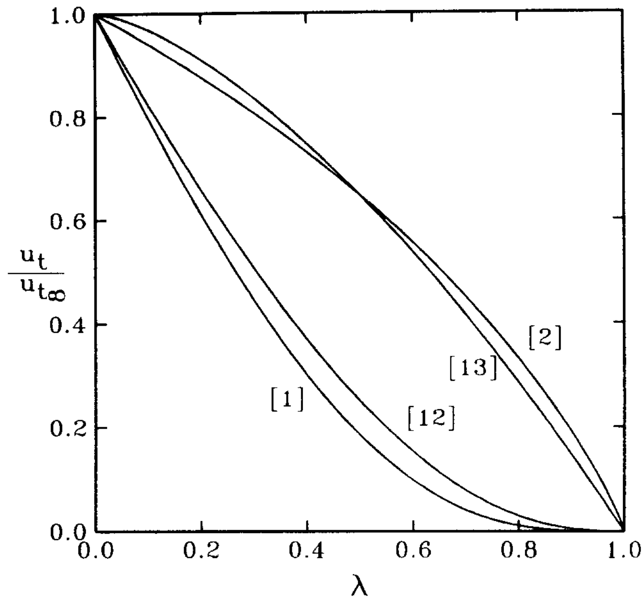

Figure 2. The ratio of bounded to unbounded terminal settling velocities for the viscous and the inertial flow regimes: predicted and empirical values.

Reynolds number with

$$\frac{2.7-\alpha}{\alpha\text{-}0.85}=0.65\mathrm{Re}_t^{0.66}. \qquad [15]$$

In [15] the coefficients 2.7 and 0.85 were derived from the result of the present analogy in the extreme flow conditions, [12] and [13], whereas the numerical coefficients on the right hand side were calculated with the utilization of a minimizing error routine.

Equation [14], with α given by [15], is plotted in figure 4 and can be compared with the Fidleris and Whitmore empirical findings, figure 1. Considering that so far we have only used widely accepted empirical relationships for the single particle drag coefficient and for the voidage function

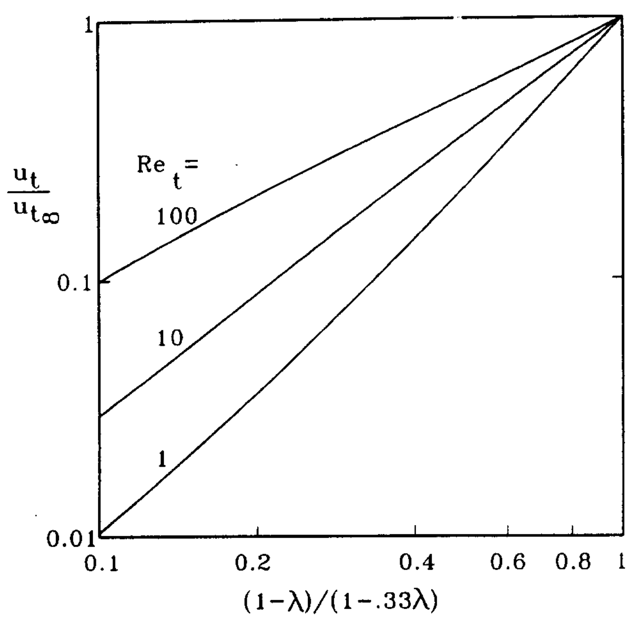

Figure 3. The ratio of bonded to unbounded terminal settling velocities as predicted by [9]–[11] for a selected value of Re_t in the intermediate flow regime.

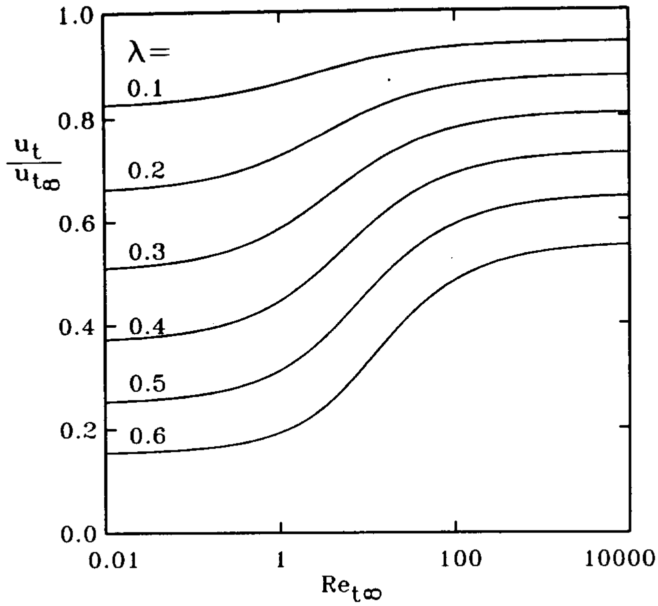

Figure 4. The ratio of bounded to unbounded terminal settling velocities as predicted by [14] and [15] for a wide range of Reynolds number and selected values of λ.

and that no adjusting factors have been introduced, the agreement between calculated and experimental values is quite satisfactory.

If one wants to use [14] for more precise evaluation of the retarding effect of the wall on the terminal settling velocity, then we can keep the same functional dependency on the parameter λ found previously and modify the coefficient α in order to fit more accurately reported measurements. For example, in this case we looked for values of α that would force [14] closer to the experimental evidence of Fidleris & Whitmore (1961). Using again a minimizing error routine we obtain

$$\frac{3.3-\alpha}{\alpha-0.85}=0.27\mathrm{Re}_t^{0.64}. \qquad [16]$$

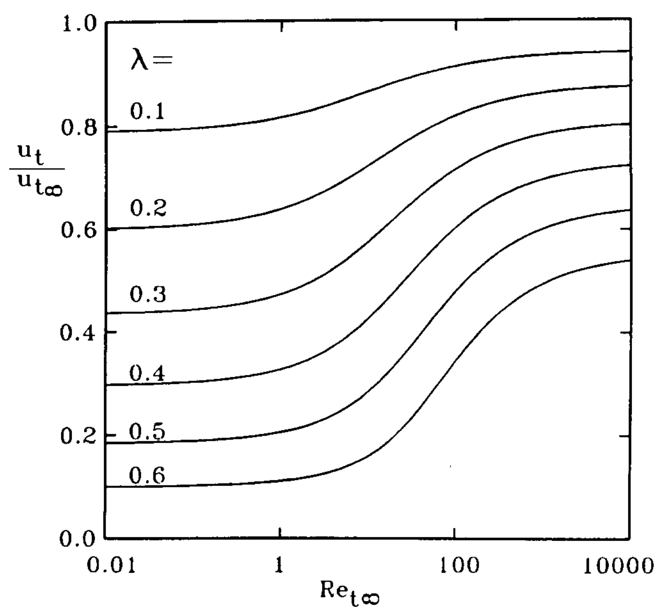

Figure 5. The ratio of bounded to unbounded terminal settling velocities as predicted by [14] and [16] for a wide range of Reynolds number and selected values of λ.

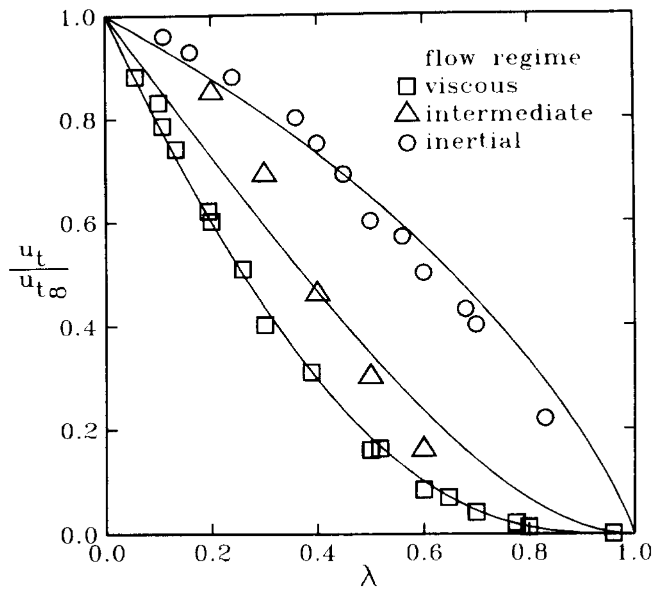

Figure 6. The ratio of bounded to unbounded terminal settling velocities as predicted by [14] and [16] for the whole range of λ and selected flow regime. Points represent reported experimental data: source of points is given in the text.

Figure 5 shows the values predicted by [14] and [16], which again should be compared with figure 1.

Finally, the potential disadvantage of this procedure, which require a trial and error routine in order to predict u_t when $u_{t\infty}$ is known from the physical parameters of the system, can be avoided by expressing α as a function of $\mathrm{Re}_{t\infty}$. For the two extreme flow regimes the value of α does not change, whereas some small adjustment is necessary in the intermediate flow regime. By using again the experimental data of Fidleris & Whitmore (1961), we obtain

$$\frac{3.3-\alpha}{\alpha-0.85}=0.1\mathrm{Re}_{t\infty}. \qquad [17]$$

The correspondent estimated values of $u_t/u_{t\infty}$ over the whole range of flow conditions are very similar to the ones depicted in figure 5 and will not be repeated.

Figure 6 depicts the predicted retarding effect of the wall on the terminal settling velocity, from [14] and [17], shown this time for the whole range of λ and for selected flow regimes. In figure 6, relationship predictions are reported for viscous, inertial and one intermediate flow regime ($\mathrm{Re}_{t\infty}=10$), and they are compared with old and newer experimental data. The experimental results of Francis (1933) and Iwaoka & Ishii (1979) are reported for the viscous flow regime, the experimental results of Munroe (1888) and Bougas & Stamatoudis (1993) for the inertial flow regime, the experimental results of Fidleris & Whitmore (1961) for the intermediate case.

The suggested relationship compares extremely well for the two extreme flow regimes and somewhat less satisfactory in the intermediate region, especially for smaller values of λ. Unfortunately, reported experimental evidence in the middle region of Reynolds number are scarce and before attempting any empirical improvement to the present relationship, as for example suggesting a functional dependence of the parameter α on the parameter λ besides that on the flow regime, more experimental support is certainly needed.

4. CONCLUSIONS

The fluid dynamic analogy between a single particle in a tube and a multiparticle suspension has enabled us to suggest a new relationship which estimates the influence of the wall on the terminal settling velocity of a single sphere in cylindrical container applicable at any Reynolds number. Small adjustment of the numerical coefficient has made the predictive capability of the relationship quite satisfactory.

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